Exercise 16

(a) What quantities are given in the problem?

(b) What is the unknown?

(c) Draw a picture of the situation for any time t.

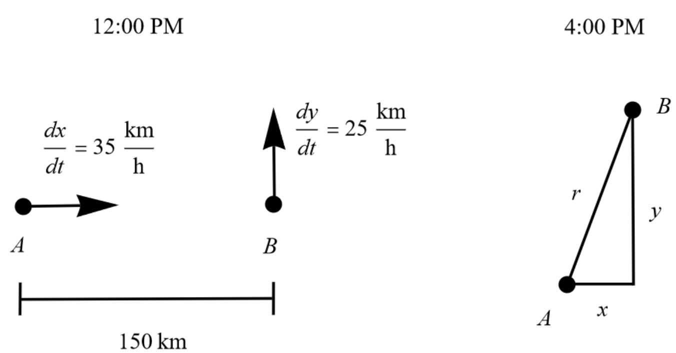
(d) Write an equation that relates the quantities.

(e) Finish solving the problem.

At noon, ship A is 150 km west of ship B. Ship A is sailing east at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 PM?

Solution

The ships' speeds, $dx/dt = 35$ km/h and 25 km/h, are known. dr/dt, the rate that the distance from ship A to ship B, after four hours is the unknown.



The Pythagorean theorem gives the relationship between the sides of the triangle.

$$r^2 = x^2 + y^2$$

$$r = \sqrt{x^2 + y^2}$$

Differentiate both sides with respect to t.

$$\begin{aligned}\frac{dr}{dt} &= \frac{1}{2}(x^2 + y^2)^{-1/2} \cdot \frac{d}{dt}(x^2 + y^2) \\ &= \frac{1}{2}(x^2 + y^2)^{-1/2} \cdot \left(2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt}\right) \\ &= \frac{1}{\sqrt{x^2 + y^2}}\left(x\frac{dx}{dt} + y\frac{dy}{dt}\right)\end{aligned}$$

Note that the sides of the triangle at 4:00 PM are $x = 150 - 35(4) = 10$ and $y = 0 + 25(4) = 100$. Therefore, the rate that the distance between the ships increases at 4:00 PM is

$$\left.\frac{dr}{dt}\right|_{\substack{x=10 \\ y=100}} = \frac{1}{\sqrt{10^2 + 100^2}}\left[10\left(35\ \frac{\text{km}}{\text{h}}\right) + 100\left(25\ \frac{\text{km}}{\text{h}}\right)\right] = \frac{285}{\sqrt{101}}\ \frac{\text{km}}{\text{h}}.$$